Spring 4-21-2022

Millikan Oil Drop Experiment

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February 6, 2022

Abstract

In this experiment we recreate the oil drop experiment conducted by Robert A. Millikan in 1910[1]. The goal was to derive the charge of the electron. We did this using a MODA01 apparatus specifically designed for this experiment. Two principal measurements were needed to derive our results. One, was the distance a drop travelled in a measured amount of time. Second, was the voltage required to keep the drop suspended in a relatively fixed position. These measurements were made for a total of 13 drops. We calculated the electron charge to be $(1.37 \pm 1.93) \times 10^{-19}$C. This is about 14% less than the accepted value of 1.602×10^{-19}C. Overall, the main sources of error arose from the distance measurements inside the oil drop chamber and the fact that the voltage on the apparatus was not able to reach down to zero.

1 Introduction

The experiment we are recreating is the oil drop experiment conducted by Robert A. Millikan in 1910[1]. The purpose of the experiment is to calculate the charge of the electron. This experiment had been done before Millikan, but he used more precise methods for both his observational process and his mathematical analysis. Because of this, the charge of the electron is, of course, known at this point. We also know of the most significant sources of error in this experiment. Namely, the viscosity of air, as it depends on the size of the drop and the pressure inside the oil drop chamber. The purpose of this paper is to contribute more data and to see the effects on uncertainty that different sources of error have on the final values.

2 Methods

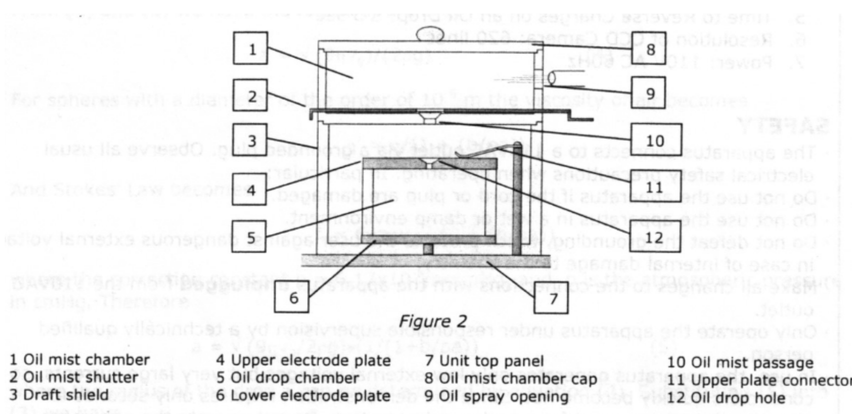

Figure 1: Schematic of the oil drop device that was ultimately connected to the MODA01 apparatus[3]

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A camera (seen in **Figure 2**) views the inside of the oil drop chamber and transfers a live video view to our oil drop computer software. The computer video was then also projected onto a large projector screen. The voltage across the oil drop chamber was controlled using a dial on the apparatus. The goal was to have an individual drop in the drop chamber to make observations simpler. For this, the oil shutter was only opened after the oil had already been sprayed in the mist chamber. For the sake of analysis, the ideal drop should be in clear view, falling slowly, and should withstand high voltages.

Our computer software superimposes a grid on the live video to measure the space inside the oil drop chamber. This grid, however, is not taken to be very accurate. To calibrate the superimposed grid, we insert a wire of known diameter

Figure 2: Image of a section of the MODA01 apparatus with the oil drop device above it. The camera is the dark device inserted into the draft shield of the oil drop device.

$(0.266 \pm 0.002\text{mm})$ into the oil drop chamber. We then measure the diameter of the wire as it appears on the video displayed on the projector screen, which was $(11.3 \pm 0.1\text{cm})$. Comparing the ratios, we are able to correct the grid spacing. A 0.25mm grid spacing, corresponded to $(8 \pm 0.1\text{cm})$ as seen on the projector screen. After the correction we concluded that the 1mm on the grid was actually $(0.76 \pm 0.01\text{mm})$

Another correction was made to the voltage displayed on the oil drop apparatus (V_a). A voltmeter was used to measure the voltage between the upper plate connector and the lower electrode plate (V_m)(see **Figure 1**). We recorded both the voltage shown by the apparatus and the voltage measured by the voltmeter for a range of 12 voltages from 2V-301V. We then plotted the voltage shown by the apparatus vs. the voltage measured by the voltmeter. The slope of the plot's best fit line (0.999) was used to derive the corrected voltage (V_{corr}) from that observed on the appartus' display thereafter. The formula used was

$$V_{\text{corr}} = \frac{V_a}{0.999}.$$

For an individual drop, we first measured the amount of time it took to fall a millimeter with no electric field inside the drop chamber (Voltage=0). The drop's fall was measured four times in order to calculate a more precise average time. After these measurements, the goal was to increase the voltage until the same drop was suspended in a relatively stable position. This voltage is recorded and then corrected. Both of these measurements were recorded for a total of 13 drops.

To calulate the velocity of a drop (v), we used the average time (t_{avg}) and the distance travelled (d)

$$v = \frac{d}{t_{\text{avg}}}$$

To calculate the radius of the drop (a), we used an equation derived from kinematics[2] using the fact that a drop is at terminal velocity (v)

$$a = \sqrt{\frac{9\eta v}{2\rho_{\text{oil}} g}}$$

Where η is the viscosity of air (1.849×10^{-5}kg/ms), ρ_{oil} is the density of the oil drop (981 ± 5kg/m^3), and g is the acceleration due to gravity (9.81m/s^2).
However, a small correction was made due to the fact that the viscosity of air is effectively different depending on the radius of a drop and atmospheric pressure (p) during the experiment (76.4 ± 0.075 cmHg). The equation used for the effective viscosity was

$$\eta_e = \eta(1 + \frac{b}{pa})^{-1}$$

This new corrected η_e was then used to recalculate a corrected radius for a drop (a_c) using the equation

$$a_c = \sqrt{\frac{9\eta_e v}{2\rho_{\text{oil}} g}}$$

This radius correction decreased the estimated radii by a maximum of about 4%.
To calculate the mass (m) of a drop, we used the formula for the volume of a sphere along with the density of the oil.

$$m = \frac{4}{3}\pi a_c^3 \rho_{\text{oil}}$$

Next, we used the Lorenz force equation

$$F = qE$$

to relate the force felt by a drop due to gravity and the force due to the electric field in the chamber. We know the voltage that was required to suspend a drop in a fixed position (V_{corr}), and we know that

$$E = \frac{V_{\text{corr}}}{d}.$$

So, to calculate the charge we set the two balancing forces equal to eachother to derive the equation

$$q = \frac{mg}{\frac{V_{\text{corr}}}{d}}.$$

This gives us the total charge on a drop. This charge is known to be quantized. In order to find the fundamental charge that these, we use an arithmetic method. This method involved taking the ratios of each calculated charge to the lowest charge.

$$r_{1l} = \frac{q_1}{q_l}, r_{2l} = \frac{q_2}{q_l}, ...$$

Then, subtracting off the whole number

$$r_{1l} = 2.14 \rightarrow 2.14 - 2 = 0.14 = r_{1w}$$

$$r_{2l} = 1.75 \rightarrow 1.75 - 1 = 0.75 = r_{2w}.$$

Next, we multiply each r_{nw} by the smallest originally used charge (q_l).

$$r_{nw}q_l = q_{no}$$

These q_{no} are the charges shown in **Table 1**. The lowest value is what we are using as the best estimate of the electron charge.

3 Results

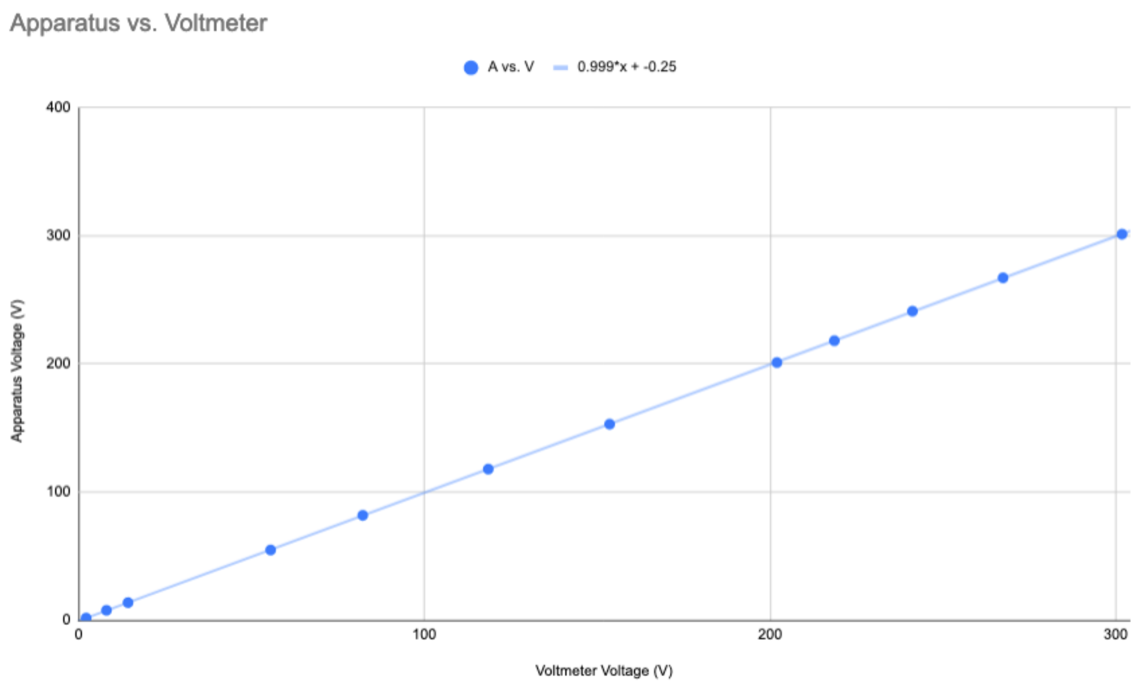

Figure 3: Plot of the voltage displayed on the apparatus vs. that on the voltmeter. The equation of the best fit line is shown above the plot, from which the slope is used to correct subsequent voltage readings from the apparatus.

This plot shows us the average factor by which the two voltages are related to eachother. For any voltage reading on the apparatus, we divide that value by the slope of the best fit line (0.999) and get what we called the ”corrected voltage”.

Table 1: This table shows the main results for each drop and their respective uncertainties. Charges shown here have not yet been reduced.

Drop	Fall Time (s)	Velocity ($\times 10^{-4}\frac{m}{s}$)	Radius ($\times 10^{-7}$m)	Mass ($\times 10^{-16}$kg)	Charge ($\times 10^{-19}$C)
1	1.65 ±0.15	4.6±0.5	19.6 ±1.1	309 ±54	36.8 ±5.14
2	3.15 ±0.15	2.4 ±0.2	14.1 ±0.6	114 ±14.7	30.1±2.23
3	1.03 ±0.15	7.4 ±1.2	24.9 ±2.1	636 ±157	281 ±62.7
4	7.23 ±0.15	1.1 ±0.1	9.2 ±0.3	31.5 ±3.53	17.2 ±0.58
5	1.99 ±0.15	3.8 ±0.4	17.8 ±0.9	231 ±36.2	43.1 ±5.01
6	1.35 ±0.15	5.6 ±0.7	21.6 ±1.4	417 ±83	101 ±17.2
7	3.69 ±0.15	2.1 ±0.2	13.0 ±0.5	89.5 ±11	93.3 ±5.93
8	4.48 ±0.15	1.7 ±0.1	11.7 ±0.5	66.4 ±7.85	36.5 ±1.93
9	2.68 ±0.15	2.8 ±0.3	15.3 ±0.7	147 ±20	55.7 ±4.83
10	2.08 ±0.15	3.7 ±0.4	17.4 ±0.9	216 ±33	158±17.6
11	3.79 ±0.15	2.0 ±0.2	12.8 ±0.5	85.9 ±10.5	63.8±3.95
12	1.96 ±0.15	3.9 ±0.4	17.9 ±0.9	236 ±37.1	123±14.5
13	5.76 ±0.15	1.3 ±0.1	10.3 ±0.4	449 ±5.12	119 ±4.99

4 Conclusions

The lowest value we derived for the charge was $(1.37 \pm 1.93) \times 10^{-19}$C. This is about 14% less than the value calculated by R. A. Millikian, which was $(1.592 \pm 0.003) \times 10^{-19}$C. We also see a signicant uncertainty in the value of our charge. One of the main sources of this uncertainty was likely from the measurement of the distance the drop travelled. We used a wire of known diameter (0.266±0.002mm) to calibrate the grid, used by the software. However, the grid as seen on the projector screen was slighty distorted vertically due to the positioning of the projector. This in turn likely decreased the apparent distance travelled and therefore increased the calculated velocity of the drops.
In terms of the best result for the charge, the largest source of error may have come from the fact that our apparatus could not reach down to 0V. Its lowest setting was about 2.2V, and this likely increased the fall time that was intended to be measured when V=0.
The drops observed were fairly large and moved relatively quickly down the chamber. A more judicious choice of drops would have benefitted this experiment.

5 References

[1] Millikan, R.A. 1911. On the elementary electrical charge and the Avogadro constant. The Physical Review, 32(4), 349-397. ; https://link.aps.org/doi/10.1103/PhysRev.2.109
[2] Hogan, Benjamin and Hasbun, Javier E. 2016. The Millikan Oil Drop Experiment: A Simulation Suitable For Classroom Use. Georgia Journal of Science, Vol. 74, No. 2, Article 7. ; http://digitalcommons.gaacademy.org/gjs/vol74/iss2/7
[3] Minschwaner, Kenneth 2022. Millikan Oil Drop Experiment: The charge of the electron. Phys 451L ; Link